Quantum theory as a tool for the description of simple psychological phenomena

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We propose the consistent statistical approach for the quantitative description of simple psychological phenomena using the methods of quantum theory of open systems (QTOS).Taking as the starting point the K. Lewin's psychological field theory we show that basic concepts of this theory can be naturally represented in the language of QTOS. In particular provided that all stimuli acting on psychological system (that is individual or group of interest) are known one can associate with these stimuli corresponding operators and after that to write down the equation for evolution of density matrix of the relevant open system which allows one to find probabilities of all possible behavior alternatives. Using the method proposed we consider in detail simple model describing such interesting psychological phenomena as cognitive dissonance and the impact of competition among group members on its unity.

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I. INTRODUCTION

It is a common opinion that in spite of various theories and impressive concrete results confirmed by numerous experiments modern psychology is still far from status of exact science such as for example theoretical physics. The main difference between these sciences is that in theoretical physics we have well defined concepts and general principles such as for example action and principle of least action in mechanics or entropy and second law in thermodynamics which let one to obtain all results of the theory by successive deductive procedure from general principles. On the other hand in psychology throughout its history many attempts were taken to bring together its concepts and facts into integral system that would allow one to describe and explain known psychological phenomena and possibly to predict some new effects. In the present paper we start from one of such theories namely psychological field theory (PFT) of Kurt Lewin and make an attempt to represent basic concepts of PFT in the language of quantum theory of open systems (QTOS). This seemingly formal representation has however indisputable advantage since allow one to use well developed mathematical methods of QTOS to analyze a variety of psychological effects and situations. In particular we show that such widely known phenomenon in psychology as cognitive dissonance(CD) can be consistently considered in the framework of the method proposed. In addition we demonstrate that this method can be used to describe group dynamics processes as well as individual behavior. The rest of the paper is organized as follows. In the Sect2 we give a brief account of information from PFT and QTOS which is necessary for understanding of the paper. In the Sect.3 which is major in the paper we realize representation of such basic concepts of the PFT as the life space,regions, locomotions and so on in the language of QTOS and introduce essential concept of density matrix of psychological system (PS) which let one to find probabilities of all possible behavior alternatives and formulate the mathematical method for description of evolution this matrix. In the Sect.4 we demonstrate the effectiveness of our approach in the framework of simple but useful model for describing several phychological phenomena such as cognitive dissonance and so forth. Now let us turn to a detailed account of the paper.

II. PREPARATORY INFORMATION

The PFT was created by K.Lewin in the middle of XX century and stated him in various books and papers (see for example [1], [2]). The comprehensive review of this theory a reader can find in the textbook of Hall and Lindzey [3]. Let us outline the main points of this theory in the form which is sufficient for understanding of the paper. The initial concept of the PFT is the concept of the life space by which K.Lewin meant a set of psychological facts (i.e. both personal incentives and external impacts connected with situation) acting at the moment on PS and determining its further evolution. For the behavior description K. Lewin proposed general but somewhat abstract formula: $B = F(S, P)$, where B means behavior, S-situation and P-person. Note in this connection that the collection of all variables describing both S and$P$ exactly constitute the life space of PS. In addition the life space of PS can be divided on separate regions each of which corresponds to a single psychological fact(for example it may be separate regions connected with job,family,game and so on)We assume that different regions are mutually disjoint.Another essential concept in PFT is a concept of a stimulus.Simple stimuli can be introduced in PFt as certain psychological forces which act on PS and stimulate it either to occupy definite region

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or to avoid it. Depending on the nature of a stimulus we can attribute to each region definite sign (plus if a stimulus is attractive and minus in opposite case). The value of a stimulus that is the tendency of PS to occupy given region or to avoid it determines the valence of corresponding region.It should be noted that in addition to simple stimuli there are also complex stimuli acting on PS of interest and composed from several simple ones.Another concept in PFT is locomotion which implies the transition (but only psychological not physical!) from certain region to another one. Note that K.Lewin proposed also simple graphic method which allows one to represent completely the current state of PS within the framework of these basic concepts. According to K. Lewin such representation is sufficient to explain actual behavior of PS in future.

However it should be noted that due to extraordinary complexity of the psychological phenomena modern psychology prefers not to talk about deterministic laws of human behavior but rather only on its statistical description.(see for example [4]). Therefore in the rest of the paper we assume the task of behavior description is solved if we can indicate the distribution function which determines probabilities of all behavior alternatives or by other words the probabilities of finding PS in arbitrary region of its life space..

Since the main goal of our paper is the representation of basic concepts of the PFT in the language of quantum theory let us remind the necessary information from QTOS. First of all as long as evolution of quantum open system is nonunitary its state should be specified with the help of density matrix (but not wave function). The main result from QTOS that we need in this paper is the Lindblad equation which describes evolution of density matrix in the case of Markov open quantum system.This equation has the next general form:

$$\frac{\partial\widehat{\rho}}{\partial t} = -\frac{i}{\hbar}\left[\widehat{H},\widehat{\rho}\right] + \sum_i \left[\widehat{R}_i\widehat{\rho},\widehat{R^+}\right] + \quad h.c, \qquad (1)$$

In Eq. (1) $\widehat{H}$ is Hermitian operator ("hamiltonian" of open system) and $\widehat{R_i}$ are non-Hermitian operators that specify all connections of open system of interest with its environment.If the initial state of the system $\rho(0)$ is known Eq. (1) allows one to find the behavior(i.e. its state at any time t and thus to determine average values of all observables relating to open system). If we are interested only in stationary states of open system we must equate r.h.s. of the Eq. (1) to zero and find stationary density matrix from this equation. Before moving on we want to explain one essential point namely : why quantum in nature Eq. (1) can be used to describe behavior of classical systems? The answer is that density matrix of quantum system represents its classical correlations as well as quantum.But information about classical correlations is contained in diagonal elements of density matrix while the information about quantum correlations in nondiagonal ones.

Thus in the situation when we are able to write closed equations including only a set of diagonal elements of density matrix and to solve them, we actually obtain classical distribution function and required description of classical analog of corresponding quantum system. As the author has shown in [5] such case holds for example in the case when all operators R_i in Eq. (1) have monomial form namely $R_i \backsim (a^+)^{k_i}(a)^{l_i}$ (where operators a and a^+ are bose operators with standard commutation rules: $\left[\widehat{a},\widehat{a^+}\right] = 1$). In this case Eq. (1) can be reduced to closed system of equations for diagonal elements of $\rho\left(\widehat{N}\right)$ (where $\widehat{N} = \widehat{a^+}\widehat{a}$ is number operator). These arguments from [5] can be extended also to the case when operators R_i are represented as monomial forms of some fermi operators f_j and $\widehat{f_k^+}$ (in the case of several degrees of freedom) and corresponding number operators $\widehat{N_i} = \widehat{f_i^+}\widehat{f_i}$ have only two eigenvalues 0 and 1. In the next part we will demonstrate as formalism of QTOS with the help of such operators can be used for the statistical description of PS systems in the language of PFT.

III. MAPPING BETWEEN PFT AND QTOS

In this part we propose the representation of all basic concepts of PFT in the language of QTOS.Note, that we will restrict ourselves to considering only such phenomena when behavior of the PS of interest is determined only by variables describing a situation while variables associated with a person play minor role.In addition we will assume that in the process of behavior there is no restructuring of the life space(that is the number of regions and all their characteristics remain fixed). Such PS we denote as "simple system". The generalization of the method proposed that takes into account also personal variables of PS will be considered elsewhere. Obviously, or PS under consideration is inside the given region of the life space or outside it. In accordance with this fact we can attribute to every region occupation number n_i which takes only two values $n_i = 1$ if PS is in the i region and $n_i = 0$ in opposite case. Let us introduce operator $\widehat{n_i}$ acting in two-dimensional linear space whose eigenvalues are 0, 1. Let us introduce also the pair of operators $\widehat{f_i^+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\widehat{f_i} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ thus that relationship $\widehat{n_i} = \widehat{f_i^+}\widehat{f_i} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ holds.To avoid misunderstanding we want to emphasize that although the operators $\widehat{f_i}$ and $\widehat{f_i^+}$ satisfy to relationship :$\widehat{f_i^+}\widehat{f_i} \ +\widehat{f_i}\widehat{f_i^+} = 1$ nevertheless studied PS systems are of course classical (not Fermi systems!). Correspodently all regions of the life space should be considered as distinguishable. Now we establish the mapping between different stimuli acting on PS of interest and corresponding operators acting on

states of relevant quantum open system. We assume that every operator R_i entering in the Lindblad equation for relevant system corresponds to certain stimulus(simple or complex) acting on PS of interest.Besides we suppose that all R_i are some monomial functions of operators f_j^+ and f_k (note that index i enumerates different stimuli acting on a PS).Note also that the number of regions can differ from number of stimuli. Let us now formulate two main correspondence rules between acting stimuli and operators R_i . Rule1: If a stimulus i is simple then we associate with it either operator $A_i = k_i \ f_i^+$ if given stimulus is attractive i.e. stimulates PS to occupy i region or operator $B_i = l_i \ f_i$ in opposite case. Coefficients k_i and l_i reflect the value of acting stimulus i. Rule2: Let a stimulus is complex (that is composed from several simple ones) then in the case when among simple stimuli i_1, i_2...are attractive and stimuli j_1, j_2...are repulsive with such complex stimulus we associate the operator $C = k \ f_{i_1} f_{i_2} ... f_{j_1}^+ f_{j_2}^+$...Note that sign of the coefficient k does not affect the final result. In addition the current state of PS can be represented by density matrix of behavior $\widehat{\rho(t)}$ in the linear space of dimension 2^N which is the tensor product $H_1 \otimes H_2 \otimes ... H_r$ $(1 \leq r \leq N)$, where N is number of different regions in the life space). Every space H_α $(\alpha = 1, ...N)$ is two-dimensional vector space with basis states $\begin{pmatrix}1\\0\end{pmatrix}_\alpha$ and $\begin{pmatrix}0\\1\end{pmatrix}_\alpha$ connected with α region.In the language of PFT the state $\begin{pmatrix}1\\0\end{pmatrix}_\alpha$ corresponds to PS which occupies α region of the life space and the state $\begin{pmatrix}0\\1\end{pmatrix}_\alpha$ corresponds to PS which is outside of it. Guided by these simple rules of correspondence one can easily represent any current situation with PS as PFT draws it using the rigorous language of QTOS. If we assume besides that considered PS has no memory then one can try to use for the description of its evolution the Lindblad equation Eq. (1) for density matrix of relevant quantum open system.(with $H = 0$). Operators R_i in this equation should be chosen of course in correspondence with the rules stated above. Thus our main assumption in the present paper is the assertion that detail describing of behavior of PS can be realized in the language of QTOS at least as well as by concepts of PFT.But now we have in hands powerful mathematical formalism which is sufficient for quantitative description of various although at present time only simple phychological phenomena. Since the previous consideration to a considerable extent was a heuristic now we want with reference to concrete psychological model to demonstrate its effectiveness. We believe that the question about applicability of the method can be solved only by careful comparision of obtained theoretical predictions relating to this and similar models and results observed in test experiments .

IV. BASIC PSYCHOLOGICAL MODEL

In this part of the paper we consider simple but useful model illustrating in our opinion the effectiveness of the method proposed. We begin by considering such well known phenomenon in psychology as cognitive dissonance (CD). According to definition (see e.g. [6]) cognitive dissonance emerges when a person has two or several ideas (cognitions) which contradict each other. The state of CD occurs for example in a situation of choice when it is impossible to estimate for sure pros and cons of different alternatives. Clearly the behavior in such sutuation will necessarily be random in its nature. Using the correspondence between PFT and QTOS specified above it is easy to formulate the situation of CD in the language of QTOS.Let us consider for simplicity the case when a person has only two positive alternatives with different incentives (which partially contradict each other) . We can represent such situation with the help of three operators R_i acting on density matrix $\widehat{\rho}$ of relevant system describing a behavior of such PS . These operators are: $\widehat{R_1} = \sqrt{\frac{a}{2}}\widehat{f_1^+}, \widehat{R_2} = \sqrt{\frac{b}{2}}\widehat{f_2^+}$ and $\widehat{R_3} = \sqrt{\frac{c}{2}}\widehat{f}_1\widehat{f}_2$. Coefficients of the operators $\widehat{R_i}$ characterisize values of correspondent stimuli in convinient normalization.Now according to our assumption (since PS of interest has no memory)its behavior can be described by the Lindblad equation for diagonal elements of density matrix which can be interpreted as distribution function for a person to make appropriate choice .

$$\frac{\partial \rho_{N_1,N_2}}{\partial t} = a\left[N_1\rho_{\bar{N}_1,N_2} - \bar{N}_1\rho_{N_1,N_2}\right] + b\left[N_2\rho_{N_1,\bar{N}_2} - \bar{N}_2\rho_{N_1,N_2}\right] + c\left[\bar{N}_1\bar{N}_2\rho_{\bar{N}_1,\bar{N}_2} - N_1N_2\rho_{N_1,N_2}\right] \tag{2}$$

where we introduce convinient notation: $\bar{N}_i \equiv 1 - N_i$ $(i = 1, 2)$. We are interesting further only in stationary solutions of Eq. (2) for which the condition $\frac{\partial \rho}{\partial t} = 0$ is satisfied. In this case matrix equation Eq. (2) taking into account the normalization condition $\sum_{N_1 N_2} \rho(N_1, N_2) = 1$ can be written in the form of next four linear equations:

$$-a\rho(0,0)-b\rho 0,0)+c\rho(1,1) = 0, \quad (3)$$
$$-a\rho(0,1)+b\rho(0,0) = 0 \quad (4)$$
$$a\rho(0,0)-b\rho(1,0) = 0 \quad (5)$$
$$a\rho(0,1)+b\rho(1,0)-c\rho(1,1) = 0 \quad (6)$$

We can easily to write down the solution of the system Eq. (3)- Eq. (6) in explicit form:

$$\rho(0,0) = \frac{abc}{\Delta_1},\ \rho(0,1)=\frac{b^2c}{\Delta_1}, \quad (7)$$
$$\rho(1,0) = \frac{a^2c}{\Delta_1},\ \rho(1,1)=\frac{ab(a+b)}{\Delta_1},$$

where $\Delta_1 = ab(a+b+c)+c\left(a^2+b^2\right)$. Note that we considered somewhat more general case then usual cognitive dissonance. Usually assumed that two competing cognitions incompatable. Evidently this case realized when fractions $\frac{a}{c}$ and $\frac{b}{c}$ tend to zero. In this case the probabilities of possible outcomes are:

$$\rho(0,0) = \frac{ab}{\Delta_2},\ \rho(0,1)=\frac{b^2}{\Delta_2}, \quad (8)$$
$$\rho(1,0) = \frac{a^2}{\Delta_2},\ \rho(1,1)=0,$$

where $\Delta_2 = a^2+b^2+ab$. If in addition we assume that cognitions 1 and 2 have identical attractiveness (the case of Buridan donkey) then the values of different alternatives are: $\rho(0,0)=\rho(0,1)=\rho(1,0)=\frac{1}{3}$ and $\rho(1,1)=0$. Thus our statistical approach results in that probability for a donkey to die of hunger is only $\frac{1}{3}$. It is worth to note also that independently from values of coefficients a,b,c for considered PS we have the relation

$$\rho(0,1)\cdot\rho(1,0)=\rho^2(0,0). \quad (9)$$

This simple relation which does not depend on parameters of the system can serve as the useful test to prove or disprove the validity of approach proposed. Now we want to demonstrate that the method proposed can be also applied for desribing simple processes of group dynamics. Let us assume we have the same mathematical model that we used for the description of cognitive dissonance but now look at it with another point of view. We believe now that this model could describe the behavior of two members of formal group which tend to reach specific personal goal or status but in their activity compete with each other .We will consider the state $\begin{pmatrix}1\\0\end{pmatrix}_i$ as the state of success and the state $\begin{pmatrix}0\\1\end{pmatrix}_i$ $(i=1,2)$ as the failure state of i member. The stationary solution of the model as before has the form Eq. (7), but now we are interesting in the another question namely :how united group will be in such process?. Explain what we are keeping in mind. Assume first that parameters of the model are connected by the relation: $c=a+b$. It is easy to see in this case that stationary density matrix Eq. (7) can be represented in the form of direct production of two matrixes: $:\rho(N_1,N_2)=\frac{1}{(a+b)^2}\cdot\begin{pmatrix}ab&0&0&0\\0&b^2&0&0\\0&0&a^2&0\\0&0&0&ab\end{pmatrix}=\rho_1\otimes\rho_2$, where $\rho_1=\frac{1}{(a+b)}\begin{pmatrix}b&0\\0&a\end{pmatrix}$ and $\rho_2=\frac{1}{a+b}\begin{pmatrix}a&0\\0&b\end{pmatrix}$. It is natural to interpret such decomposition as the desintegration of the group. Our main task now is to introduce a quantity which could measure the unity of the group, i.e. how far is the group from state of desintegration. For this purpose we will use the analogy of this problem with a similar problem in quantum theory of composite systems. Remind that in theory of quantum entanglement to measure how far is given pure state of composite system from factorized one it is convinient to use the quantity which called concurrence [7]. In particular if we are interesting only in two qubit pure states which can be represented in the next vector form $|\Psi\rangle \equiv \left| z_1,\ z_2,\ z_3,\ z_4 \right\rangle$ (with normalization condition $|z_1|^2+|z_2|^2+|z_3|^2+|z_4|^2=1$) the concurrence (in our case we prefer to call corresponding quantity as "unity ") can be defined as $U=2|z_1z_4-z_2z_3|$. It is easy to prove that for all two qubit states $0\le U\le 1$. Besides $U=0$ for factorized states and $U=1$ for maximally entangled (for example four Bell's states). In our problem however all diagonal elements of density matrix are real moreover they are positive.Therefore there are no explicit analogs of Bell's states for our system of interest which is of course classical.Nevertheless the concept of group unity (i.e. classical analog of concurrence) is still very useful. We are interested here how group unity depends on parameters of the model.

According to definition, group unity $U = 2\left|\frac{a^2b^2c(a+b-c)}{[c(a^2+b^2+ab)+ab(a+b)]^2}\right|$. We consider coefficients a,b in this expression as fixed parameters and c as control parameter and we are interested in what is the effect of competition on group unity.We can find the optimal value of c from the condition of maximum U: $\frac{\partial U}{\partial c}=0$ which implies $c_{op}=\frac{ab(a+b)}{a^2+b^2+3ab}$. Substituting this value c_{op} in U we obtain that $U_{\max}=\frac{ab}{2(a+b)^2}$. But it should be note that when c tends to infinity the corresponding value of U asymptotically tends to $U_\infty=\frac{2a^2b^2}{(a^2+b^2+ab)^2}$. Therefore we must compare two values : $U_{\max}$ and U_∞.

It is easy to see that this problem reduced to the evaluation of the expression: $F(a,b)=\left(a^2+b^2+ab\right)^2-4ab(a+b)^2$. Condition$F(a,b)\ge 0$ implies that $U_{\max}\ge U_\infty$ and vice versa. Let $b\equiv ta$, then $F(a,b)\equiv a^2f(t) = a^2\left[\left(1+t+t^2\right)^2-4(1+t)^2\right]$.

The equation $f(t) = 0$ has two positive roots t_1, t_2 (connected by relation $t_1 t_2 = 1$) which can be found from the solution of quadratic equation :$t + \frac{1}{t} = 1 + 2\sqrt{2}$. Rough numerical values of roots are: $t_1 \approx 0,3$, $t_2 \approx 3,3$. and thus we obtain that $F(a,b) \leq 0$ in interval $t_1 \leq t \leq t_2$).

It is interesting to note that starting from very simple virtually toy model we nevertheless come to the conclusion which was not obvious in advance. It turns out that when abilities or efforts of two competing members to achieve some goal differ not very essentially strong competition can increase the unity of the group. In opposite case when the difference is significant it is necessary to establish the optimal level of competition to get the maximal unity of the group.

Let us sum up the results of our study. Starting from ideas of PFT and QTOS we established the connection between these two theories that seemed far from each other and proposed the consistent approach for describing simple phychological phenomena relating both to individuals and groups of individuals.In the framework of concrete simple model we have demonstrated the effectiveness of the method proposed to calculate the probabilities of behavior alternatives and also to predict some peculiarities of behavior which can hardly be revealed by other approaches. We express the hope that further development of the method let one to extend its applicability to more broad sphere of interesting psychological phenomena.

[1] K. Lewin , A Dynamical theory of Personality, New York, McGraw-Hill, 1935.

[2] K. Lewin, Principles of Topological Psychology, New York, McGraw-Hill,1936.

[3] C. S. Hall, G. Lindzey, Theories of Personality, 2ed, J.Wiley&Sons, New York,1970.

[4] L. Ross, R. E. Nisbett, The Person and the Situation, McGraw-Hill, New York,1991..

[5] E. D. Vol, quant-ph/1107.0548, (2011).

[6] E. Aronson, The Social Animal, 7ed, W. H. Freeman and Company, New York,1995.

[7] W. K. Wootters, Phys. Rev. Lett. **80**, 2245, (1998).